STUDENTS' IDEAS ABOUT MATHEMATICS

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This study examined students' attitudes to and ideas about mathematics at the beginning of a mathematics or engineering degree at university. The instrument used was one of Schoenfeld (1989) considerably modified for use with university students. Seventy-four students took part in the study.
Several interesting results were noted. Both male and female students attributed their success in mathematics to intrinsic values such as hard work. There was a significant difference between the ideas of students who had studied 3 unit mathematics and those who had studied 4 unit (the highest level in NSW). Students who had studied 3 unit described school mathematics as "just memorising". However 86% of the students described mathematics as "interesting" and gave that as a reason for doing mathematics.
The most interesting part of the study was the answers to 10 open-ended questions. Here, this group of students demonstrated a mature understanding of the links between areas of mathematics, of everyday applications of what they had studied at school and were at ease with discussing their feelings about mathematics and learning mathematics. The mathematics they had studied so far was widely perceived as useful in the development of personal skills - teaching clear and logical thinking, fostering persistence, patience and discipline.

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The aim of the study was to provide a basis for comparison of students' ideas of mathematics at the beginning and end of their degree and aim to encourage them to reflect on their previous learning experiences. The study will not be complete until 1995 however results of the initial survey are of interest and are presented here.

These results will form part of the information used for the design of future curriculum. Parallel studies of language used in lectures by academic staff and of student responses to language-based assessment in mathematics will contribute to curriculum change.

METHOD

Subjects

The subjects were 74 first-year university students at the University of Technology, Sydney. These were made up of 46 students from the Bachelor of Science degree in mathematics (64% of cohort) and 28 from a tutorial group of 30 from the Bachelor of Engineering (Electrical).

The subjects were predominantly a young group straight out of school with 75% aged 17-19 and 19% aged 20-24. There were 49 male and 25 female students. There are more males because of the inclusion of Engineering students - in the BSc (mathematics) the numbers of males and females are approximately equal. 78% had completed 3 or 4 unit mathematics in the NSW Higher School Certificate.

The Instrument

The instrument is based on one designed by Schoenfeld (1989). Many of the questions were changed to language more suitable for older Australian students. Other questions did not apply in the Australian context and were deleted. Some questions were added to meet our needs (see appendix for instrument).

Procedures

The surveys were distributed in the first weeks of semester just after the students arrived at university. It is therefore a survey of their ideas regarding school mathematics rather than a survey of ideas regarding university mathematics.

All students, 102 in total, comprising 72 from the BSc (Mathematics) and 30 from BE (Electrical) were given the opportunity to participate and invited to take the survey home and complete it at their leisure. The surveys were anonymous. 74 surveys were returned. From the detailed answers given to the open-ended questions, students took considerable time and care with their answers.

DISCUSSION OF RESULTS

Attributions of success or failure

This group of students attributed their success in mathematics to intrinsic factors - primarily to hard work (97%) and to a lesser extent, natural ability (70%). The idea of teacher bias was strongly rejected. Eighteen percent felt that luck played some role but in general this idea was also rejected.

Doing badly in maths was also strongly attributed to intrinsic factors - not studying hard enough (92%) and careless mistakes (90%). Interestingly lack of ability was not seen as a significant factor in explaining lack of success (11%).

It appears that the students felt that they had some natural ability in mathematics which can explain some of their success, but are aware that hard work and attention to accuracy are the major determinants of their performance. This certainly sounds like an ideal group to do a degree in mathematics or engineering!

The attributions of success did not vary significantly between mathematics and engineering students nor was there a significant difference between male and female students. The results on sex differences (or lack of them) replicate Schoenfeld's (1989) results. In fact, no sex differences were noted on any question so results have been quoted as totals.

Perceptions of School Mathematics

Of the 74 students, 57 had studied either 3 or 4 unit mathematics for the 1991 NSW Higher School Certificate, two had studied 2 unit and fourteen had not sat for the 1991 HSC (mature aged or overseas students). Overall, three-quarters of the students felt that school maths was "mostly facts and procedures that have to be memorised", although a similar proportion also felt that school maths was "thought provoking".

When given the chance to expand on the idea of memorising in the open-ended questions, students seemed to be aware of the advantages and disadvantages of memorising and so this perception of memorising must be tempered with the fact that these students have recently completed the Higher School Certificate exam where speed is essential.

There was a significant difference between 3 and 4 unit students ($p < 0.02$), with 3 unit students more inclined to classify school maths as just memorising.

Views on Mathematics and English

There was a strong feeling that natural ability plays an important role in mathematics and English. 86% felt it was true that some people are good at maths and some are not and 82% felt the same about

English. Further, almost two-thirds of students felt there was no connection between being good at English and being good at mathematics.

Most students (86%) believed that in mathematics something is either right or wrong, whereas this is not considered true for English (14%). This differed from Schoenfeld's (1989) results where he found that students found the right/wrong distinction sharper in English classes. He attributed this to the classroom practice where part marks are given for incorrect mathematics answers but English grammatical and spelling errors are emphasised by `ugly red marks.'

There were several questions comparing features of the teaching of mathematics and English. Students considered it desirable that both English and mathematics teachers show students a variety of ways to look at the same question (86% and 96% respectively) but whereas showing you the exact way to answer test questions was not considered important for English teachers (21%) nearly half of the students (44%) felt that it had some importance for Mathematics teachers.

Perceptions of Mathematics

There were some encouraging results in this section of the questionnaire. Most students (71%) felt that everything important is not already known by mathematicians, that in mathematics you can be creative and discover things by yourself (86%) and that it is not true that maths problems can be done correctly in only one way (92%). This contrasts with students perceptions of school mathematics which most considered was mostly facts and procedures that have to be memorised.

Reasons for Learning Mathematics

The strongest reasons for learning mathematics were wanting to do well in the course, because it will be useful in getting a job, and because it is required for the student's program - all practical considerations. However of equal importance was that mathematics is interesting, with 86% quoting this as a reason for learning mathematics, and no student disagreeing strongly.

Open Ended Questions

Many respondents to the survey gave surprisingly detailed comments showing that they had given considerable thought to their answers and that they appreciated the opportunity to voice their opinions.

(Question: Do you think mathematicians work alone on problems or together? Which do you think is better and why?) There was a strong preference for mathematicians to work at least part of the time in groups, to share ideas and gain different perspectives. *(Question: Are the different mathematics courses you have taken related to each other in any way? If so, how?)* Most students perceived the different mathematics courses (algebra, geometry, trigonometry) as related, with particular mention being made of algebra as a link between courses, and of basic problem solving techniques as common to all courses. *(Question: Why do you think so much time in geometry classes is spent learning to write proofs?)* Learning to write proofs in geometry was seen as important in aiding understanding of the theory, in improving analytical skills, and in learning that you must justify your answers. This last attribute was seen as fundamental in real-life problems particularly in engineering.

(Question: How much of you ability to do proofs shows up when you take maths tests?) Students varied their assessment of how much of their ability to do mathematics showed up in maths tests, with opinions evenly split between "most," "some" and "not much". One student questioned the meaning of "doing maths" - "What is meant by doing maths? If doing mathematics means pressing the keys of the calculators, speed of writing and time of thinking, and then paper work free of `silly' mistakes then during any maths test I realise I am unable to do maths."

(Question: What can you do if you get stuck while doing a maths problem?) Various strategies for coping when you get stuck while doing a maths problem were suggested, ranging from "keep on working" to

"panic". The most common strategies were to leave the question and come back later, approach it from a different angle, or seek for help from notes, friends, tutors, lecturers. There may have been some confusion over the interpretation of this question as to whether the "getting stuck" was under exam conditions or not.

(Question: In what way, if any, is the maths you have studied useful?) Responses to the question on the usefulness of mathematics fell into two categories - external (relevance) and internal (personal development). Mathematics was seen as relevant to problem solving in work, particularly engineering and computing and to everyday life. "I use maths all the time - working out my economy rate when bowling in cricket, working out at which angle to hit the ball when playing snooker, counting my money, working out my weight index so that I don't get too fat etc". And somewhat more succinctly "it means the puzzles are easier". Mathematics studied so far was also widely perceived as useful in the development of personal skills - teaching clear and logical thinking, fostering persistence, patience and discipline.

(Question: Do you think students can discover mathematics on their own, or does mathematics have to be shown to them? Please explain.) Despite most students indicating in the questionnaire (Q24) that in maths you can be creative, many revealed a lack of confidence when asked whether students can discover mathematics on their own. The most common response was that students need to be shown the basic rules, but can discover maths on their own, given motivation, dedication and guidance along the way. Some students felt that self-discovery of mathematics was only for the very smart, while a number felt it was a waste of time. "After all, it took mathematicians years to work out some of the stuff". "Teachers are more successful at teaching new ideas rather than trying to follow it in the text book".

(Question: If you understand the material, how long should it take to solve a typical homework problem? What is a reasonable amount of time to work on a problem before you know it is impossible?) Although most students felt that a typical homework problem should take less than ten minutes, opinions varied widely as to what is a reasonable amount of time to work on a problem before you know it is impossible. A few students are apparently able to assess whether a question is impossible within two to three minutes, most will work for between ten minutes and one hour; and a few very determined students are prepared to work indefinitely. "I don't stop working on something until someone says it is impossible and then I usually try to prove them wrong." "A problem is impossible when you have exhausted all avenues available".

(Question: How can you know whether you understand something in maths? What do you do to measure (test) yourself?) Again a number of methods for knowing whether you understand something in maths were suggested. Most popular was simply when you get your problems correct, or can solve them without reference to textbook or notes. Also mentioned was being able to follow the theory and apply it in different situations; when you actually start enjoying the work; and being able to explain it to others. "If I can explain it to someone else I know that I understand it myself". Most students tested themselves with extra questions.

(Question: How important is memorising in learning mathematics? If anything else is important, please explain how.) Finally there was strong agreement that while memorising formulas is important in that it can save you time, understanding is vital. "Memorising becomes less important as understanding is achieved". "Memorising (formulae) has its role but understanding how to use them is the key".

IMPLICATIONS FOR TEACHING

There are several implications for teaching from this study. The strong preference for mathematicians to work at least part of the time in groups suggests greater use of group activities in mathematics learning and assessment. Being able to explain the work to others as an aid to personal understanding suggests part of the assessment of mathematics students could be the presentation of short seminars to class groups. The fact that many students expressed a preference for teachers rather than books reinforces the importance of backup services such as mathematics study centres, peer teaching or after school homework sessions. Students are looking for personal contact in their mathematics learning.

There are also implications for the teaching of university mathematics. Students are willing to work hard, they can make connections between branches of mathematics, most display a mature ability to reflect on their learning. This is a good basis on which to start a university career. We need to ensure that the overloaded curricula and the types of assessment procedures we use are changed so that students are not forced into memorising mode. Our survey has shown that students are willing to articulate their reflections on school mathematics and I believe this can be translated into reflection on the conceptual difficulties in early university calculus as identified in Tall (1992) provided students are given an opportunity. Students in this survey showed a lack of confidence in their ability to discover mathematics on their own - again care must be taken to give students confidence in their abilities.

As one of my students said to me last week "You don't want me to just learn this, you want me to think!"

CONCLUSION

Several interesting results were noted. Both male and female students attributed their success in mathematics to intrinsic values such as hard work. There was a significant difference between the ideas of students who had studied 3 unit mathematics and those who had studied 4 unit (the highest level in NSW). Students who had studied 3 unit described school mathematics as "just memorising". However most of the students described mathematics as "interesting" and gave that as a reason for doing mathematics. The mathematics they had studied so far was widely perceived as useful in the development of personal skills - teaching clear and logical thinking, fostering persistence, patience and discipline.

There were difficulties with students conceptions of `impossible' with most students believing that a problem was impossible if they spent ten minutes to an hour working on it. This was a longer time span than that shown in Schoenfeld's study and may represent the older age group of the students surveyed. One would expect the idea of a problem being `impossible' to change over their course of study at university.

The students involved in this study were enrolling in either a mathematics degree or an engineering degree, both of which clearly require a high level of competence in mathematics. However many other degrees, such as Business and Psychology, include either mathematical or statistical subjects. The students enrolling in these courses have a more varied background in mathematical experience and a comparison of their attitudes to and ideas about mathematics with those of our original survey group will also prove interesting.

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